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EE 381: Electronic Properties of Materials
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Homework 6

Part A

Problem: Long coaxial cable with an inner conductor of radius R_1, an insulating region from R_1 to R_2, and a return conductor extending from R_2 to R_3. The inner conductor carries current $+I$ and the outer conductor carries current $-I$. By symmetry the magnetizing field has only an azimuthal component $H_\varphi(r)$. Ampere's law gives

$$\oint \mathbf{H} \cdot d\boldsymbol{\ell} = H_\varphi(r)(2\pi r) = I_{\text{enc}}(r).$$

Region 1: $0 < r < R_1$. The current density in the inner conductor is

$$J_1 = \frac{I}{\pi R_1^2}.$$

The enclosed current is

$$I_{\text{enc}}(r) = J_1 \pi r^2 = I \frac{r^2}{R_1^2}.$$

Thus

$$H_\varphi(r) = \frac{I}{2\pi} \frac{r}{R_1^2}, \qquad 0 < r < R_1.$$

Region 2: $R_1 < r < R_2$. The loop encloses the full inner current and none of the return current, so we have

$$I_{\text{enc}}(r) = I.$$

Hence

$$H_\varphi(r) = \frac{I}{2\pi r}, \qquad R_1 < r < R_2.$$

Region 3: $R_2 < r < R_3$. The return current is uniformly distributed with a density of

$$J_2 = \frac{-I}{\pi(R_3^2 - R_2^2)}.$$

The portion enclosed by a loop of radius r is

$$I_{\text{ret,enc}}(r) = J_2 \pi (r^2 - R_2^2) = -I \frac{r^2 - R_2^2}{R_3^2 - R_2^2}.$$

And the total enclosed current is

$$I_{\text{enc}}(r) = I + I_{\text{ret,enc}}(r) = I \frac{R_3^2 - r^2}{R_3^2 - R_2^2}.$$

Thus

$$H_\varphi(r) = \frac{I}{2\pi r} \frac{R_3^2 - r^2}{R_3^2 - R_2^2}, \qquad R_2 < r < R_3.$$

Region 4: $r > R_3$. In in region 4, we hav that both currents are enclosed:

$$I_{\text{enc}}(r) = I + (-I) = 0.$$

Thus

$$H_\varphi(r) = 0, \qquad r > R_3.$$

Part B

In the ideal model, the net enclosed current outside the cable is zero, giving $H = 0$. However, I think there are several imperfections that could lead to leakage of the magnetic field:

1) The inner conductor may not be perfectly centered, breaking the cylindrical symmetry.
2) Connectors, bends, and non-uniformities geometries distort the current distribution, producing external fields
3) At higher frequencies, the skin effect would cause non-uniform current distributions, and thus, would prevent the perfect cancellation
4) Nearby conductive or magnetic materials may alter current paths