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EE 381: Electronic Properties of Materials
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Homework 4

Problem 1

Question 1. (a) The critical plasma frequency f_p of the ionosphere is related to free electron density N_e as

$$f_p = 8.98 \times 10^{-3}\sqrt{N_e} \quad [\text{Hz}, N_e \text{ in m}^{-3}].$$

Thus,

$$N_e = \left(\frac{f_p}{8.98 \times 10^{-3}}\right)^2.$$

Given the foF2 map (MHz), each color-coded value can be turned into electron density. For example:

$$f_{oF2} = 9\,\text{MHz} \quad \Rightarrow \quad N_e \approx \left(\frac{9 \times 10^6}{8.98 \times 10^{-3}}\right)^2 \approx 1.0 \times 10^{12}\,\text{m}^{-3}.$$

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% Convert foF2 colorbar values (in MHz) to electron density (m^-3)
foF2_MHz = 2:0.5:14;
fp_Hz = foF2_MHz*1e6;
Ne_m3 = (fp_Hz/8.98).^2;
plot(foF2_MHz, Ne_m3/1e11, 'o-','LineWidth',1.2);
xlabel('foF2 [MHz]'); ylabel('N_e [10^{11} m^{-3}]'); grid on;
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Question 2. (b) Daylight raises ionization → foF2 increases over sunlit regions. At 9 MHz, large daytime regions will have foF2 greater than 9 MHz, so the 9 MHz signals will reflect instead of penetrating, so that an uplink to a satellite is unreliable during daytime because the ionosphere behaves like a mirror at that frequency.

Problem 2

Question 3. (a) The plot of the optical constants of gold (refractive index n and extinction coefficient k) shows that for photon energies below 2 eV:

- n is relatively small (~ 0.2–1.2), while k is large (~ 12 at 0.6 eV, decreasing toward 1 near 2 eV).
- The dielectric function is
$$\varepsilon = (n + ik)^2 = (n^2 - k^2) + i(2nk).$$
Because $k^2 \gg n^2$ in this regime, ε' is negative (metal-like), while ε'' is large at low energies and decreases toward 2 eV.
- Qualitatively, ε' rises from about -150 at 0.6 eV toward -2 near 2 eV, while ε'' decreases from ~ 30 to ~ 1.

These trends are characteristic of a Drude metal. Fitting the Drude model,
$$\varepsilon(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\gamma\omega},$$
to the low-energy data gives:
$$\hbar\omega_p \approx 9\,\text{eV}, \qquad f_1 = \frac{\omega_p}{2\pi} \approx 2.2 \times 10^{15}\,\text{Hz},$$
$$\hbar\gamma \approx 0.07\,\text{eV}, \qquad f_2 = \frac{\gamma}{2\pi} \approx 1.7 \times 10^{13}\,\text{Hz}.$$

Question 4. (b) The plasma frequency gives the conduction electron density:
$$n_e = \frac{\varepsilon_0 m_e \omega_p^2}{e^2} \approx 6 \times 10^{28}\,\text{m}^{-3},$$
The conductivity can be estimated from
$$\sigma \approx \frac{\varepsilon_0 \omega_p^2}{\gamma} \approx 4 \times 10^7\,\text{S/m},$$

the idea is to try to fit the function we found to the actual data.