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EE 381: Electronic Properties of Materials
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Homework 2

Problem 1: Classical Conductivity Model for Copper

The electrical conductivity of copper at room temperature is $\sigma \approx 6.2 \times 10^7$ S/m.

Question 1. (a) Using copper's density and assuming one free electron per atom, find the average scattering time τ. Number density:

$$n = \frac{\rho}{M} N_A \approx \frac{8.96 \times 10^3}{0.063546}(6.022 \times 10^{23}) \approx 8.49 \times 10^{28} \text{ m}^{-3}.$$

From Drude:

$$\sigma = \frac{ne^2\tau}{m_e} \quad \Rightarrow \quad \tau = \frac{m_e \sigma}{ne^2}.$$

$$\boxed{\tau \approx 2.6 \times 10^{-14} \text{ s}}$$

Question 2. (b) The average energy of conducting electrons in copper is about 7 eV. What is their average speed?

$$\frac{1}{2} m_e v^2 = E \quad \Rightarrow \quad v = \sqrt{\frac{2E}{m_e}}.$$

With $E = 7(1.602 \times 10^{-19})$ J,

$$\boxed{v \approx 1.6 \times 10^6 \text{ m/s}}$$

Question 3. (c) Average number of atomic distances between scatterings. Mean free path:

$$\ell = v\tau \approx (1.6 \times 10^6)(2.6 \times 10^{-14}) \approx 4.1 \times 10^{-8} \text{ m} = 41 \text{ nm}.$$

Atomic spacing: $a \approx 2.6 \times 10^{-10}$ m.

$$\ell/a \approx 1.6 \times 10^2.$$

Thus electrons travel about 160 atomic spacings before scattering. For times $\gg \tau$, with random scattering, the average velocity is

$$\boxed{\langle \vec{v} \rangle = 0}$$

while the average speed is still $\sim 1.6 \times 10^6$ m/s.

Problem 2: Drift Velocity

Question 4. (a) Drift velocity for 100 V across 1 cm copper wire. Electric field: $E = V/L = 100/0.01 = 10^4$ V/m.

Drift velocity:

$$v_d = \frac{\sigma E}{ne} \approx \frac{6.2 \times 10^7 \cdot 10^4}{(8.49 \times 10^{28})(1.602 \times 10^{-19})}.$$

$$\boxed{v_d \approx 46 \text{ m/s}}$$

Comparison: $v_d \ll v$. Ratio $v/v_d \sim 3.4 \times 10^4$.

Question 5. (b) Is drift velocity equal to signal velocity? No. Drift velocity is very slow (tens of m/s). The information velocity in a wire is the propagation of the electromagnetic field, which travels at a large fraction of c.

Problem 3: 1D Quantum Picture (parabolic band, $m^* = m_e$)

Given: $N = 8.5 \times 10^{28}$ m^{-3}, $E_F = 7$ eV above the band minimum, $m^* = m_e$, mean-free time $\tau = 10^{-14}$ s. A wire of length $L = 1$ cm has $V = 100$ V applied. Take $e = 1.602 \times 10^{-19}$ C and $m_e = 9.109 \times 10^{-31}$ kg.

Question 6. (a) Electric field inside the wire.

$$E = \frac{V}{L} = \frac{100}{0.01} = \boxed{1.0 \times 10^4 \text{ V/m}}.$$

Question 7. (b) Force on each electron due to the field.

$$\vec{F} = -e\,\vec{E}, \qquad |F| = eE = (1.602 \times 10^{-19})(10^4) = \boxed{1.602 \times 10^{-15} \text{ N}}$$

(opposite the field).

Question 8. (c) Change of momentum over one scattering time.

$$\Delta p = F\,\tau = (1.602 \times 10^{-15})(10^{-14}) = \boxed{1.602 \times 10^{-29} \text{ kg} \cdot \text{m/s}}.$$

Question 9. (d) Change of energy and its fraction of the Fermi energy. Near the Fermi surface with parabolic dispersion $E = \frac{p^2}{2m_e}$,

$$v_F = \sqrt{\frac{2E_F}{m_e}} = \sqrt{\frac{2 \cdot 7\,\text{eV}}{m_e}} \approx 1.57 \times 10^6 \text{ m/s}.$$

A small shift Δp changes the kinetic energy by $\Delta E \approx v_F\,\Delta p$:

$$\Delta E \approx (1.57 \times 10^6)(1.602 \times 10^{-29}) = 2.51 \times 10^{-23} \text{ J} = \boxed{1.57 \times 10^{-4} \text{ eV}}.$$

Fraction of E_F:

$$\boxed{\Delta E/E_F \approx 2.24 \times 10^{-5}}.$$

Question 10. (e) Average drift velocity due to the applied field. $v_d = \dfrac{eE\tau}{m_e} = \dfrac{\Delta p}{m_e}$:

$$\boxed{v_d \approx \frac{1.602 \times 10^{-29}}{9.109 \times 10^{-31}} = 1.76 \times 10^1 \text{ m/s}} \quad \text{(about 17.6 m/s)}.$$

Question 11. (f) Current density using the DOS near E_F. Only electrons within ΔE of E_F are driven out of equilibrium. With $g(E_F) \approx 10^{28}$ m^{-3} eV^{-1}, the density is

$$n_{\text{part}} \approx g(E_F)\,\Delta E = (10^{28})(1.57 \times 10^{-4}) = 1.57 \times 10^{24} \text{ m}^{-3}.$$

Thus

$$J \approx n_{\text{part}}\,e\,v_d = (1.57 \times 10^{24})(1.602 \times 10^{-19})(17.6) \approx \boxed{4.4 \times 10^6 \text{ A/m}^2}.$$

1/3 (it's 3D)