EE 359, Pset #1

2.1) (b)

$T[x[n]] = \sum_{k=n_0}^{n} x[k] \quad n \neq 0$

memoryless since $y[n] = T[x[n]]$ only depends on nth value...

→ not stable : $|x[n]| < \epsilon \Rightarrow T[x[n]] \leq \sum_{k=n_0}^{n} |x[k]|$

$\leq |n - n_0| M$

& as $n \to \infty$, $T \to \infty \Rightarrow$ not stable

→ not causal (on future values when $n < n_0$)

→ linear ✓

$\Rightarrow T[a x_1[n] + b x_2[n]] = a \sum_{n_0}^{n} x_1[n] + b \sum_{k=n_0}^{n} x_2[n]$

$= a T[x_1[n]] + b T[x_2[n]]$

→ not time invariant since

$T[x[n-n_0]] = \sum^{n-n_0} x[k] \neq y[n-n_0] = \sum_{n_0}^{n-n_0} x[k]$

(c) stable : $|T[x[n]]| \leq \sum_{k=n-n_0}^{n+n_0} |x[k]| \leq \sum_{k=n-n_0}^{n+n_0} x[k] M \leq [2n_0 + 1] M$

for all $|x[n]| < M$ ✓

Not causal (depends on future values of $x[n]$)

linear ✓ → $\sum_{k=n-n_0}^{n+n_0} a x_1[k] + b x_2[k]$ → linearity of summation...

$= a T[x_1[n]] + b T[x_2[n]]$

Time-invariant $\Rightarrow T[x[n-n_0]] = \sum_{k=n_0}^{n} x[k-n_0]$

$= \sum_{k=0}^{n-n_0} x[k] \neq y[n-n_0] = \sum_{k=n_0}^{n-n_0} x[k]$

→ Not memoryless since Not causal

(e) $T(x[n]) = e^{x[n]}$

→ stable since $|x[n]| < M \rightarrow T[x[n]] = |e^{x[n]}|$
$\leq e^{|M|}$ → stable

→ yes, it is causal ✓

→ not linear (exponent adding → multiply in e)

→ time-invariant since
$T[x(n-n_0)] = e^{x[n-n_0]} = y[n-n_0]$ ✓

→ yes, since only depends on n^{th} value of x

(g) $T(x[n]) = x[-n]$
→ stable since $|T(x(n))| \leq |x[n]| \leq M$.
→ Not causal (depends on $-n$)
→ yes, linear since
$T(a x_1[n] + b x_2[n]) = a T(x_1[n]) + b T(x_2[n])$

→ Not time invariant:
$T(x[n-n_0]) = x[-n-n_0]$
$\neq y[n-n_0] = x[-n+n_0]$
→ not memoryless, since not causal

(h) $T[x[n]] = x[n] + 3u[n+1]$

→ stable since for $n \geq -1$, $|T[x[n]]| \leq M+3$, and $\leq M$ for $n < -1$

→ causal since no dependence of future n

→ not linear:

$T[ax_1[n] + bx_2[n]] = ax_1[n] + bx_2[n] + 3u[n+1]$
$\neq ax_1[n] + bx_2[n] + 6u[n+1]$

→ not time invariant: $T[x[n-n_0]] = x[n-n_0] + 3u[n+1]$
$= y[n-n_0] = x[n-n_0] + 3u[n-n_0+1]$

→ memoryless → only depends on nth value of x only...

2.23)

(a) $T[x[n]] = \cos(\pi n)x[n]$ ⇒ equal to $(-1)^n x[n]$

→ stable since cos only takes value from -1 to 1, if $|x[n]| \leq M$, then $T \leq |M|$ as well

→ causal because only depends on current n

→ linear since let $T[ax_1[n] + bx_2[n]]$
→ $\cos(\pi n)(ax_1[n] + bx_2[n]) = ay_1[n] + by_2[n]$

→ not time invariant
if $y[n] = T[x[n]] = (-1)^n x[n]$, then
$T[x[n-1]] = (-1)^n x[n-1] \neq y[n-1]$

(b) $T[x[n]] = x[n^2]$

→ stable since if x is bounded, $x[n^2]$ is bounded as well

→ not causal (future values)

→ linear since

$$T(ax_1[n] + bx_2[n]) = ax_1[n^2] + bx_2[n^2]$$

$$= ay_1[n] + by_2[n]$$

→ not time-invariant since

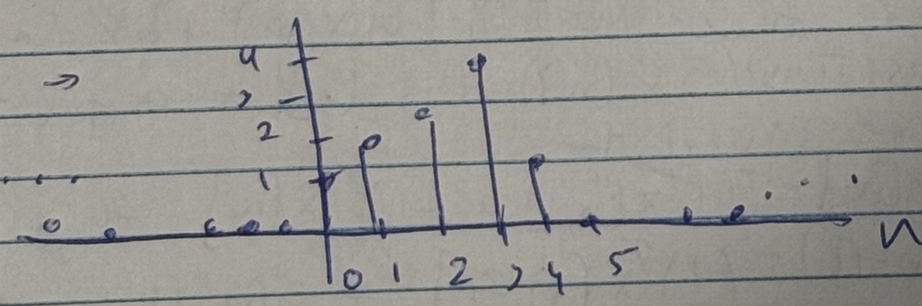
if $y[n] = T(x[n]) = x[n^2]$, then

$$T(x[n-1]) = x[n^2-1] \neq y[n-1]$$

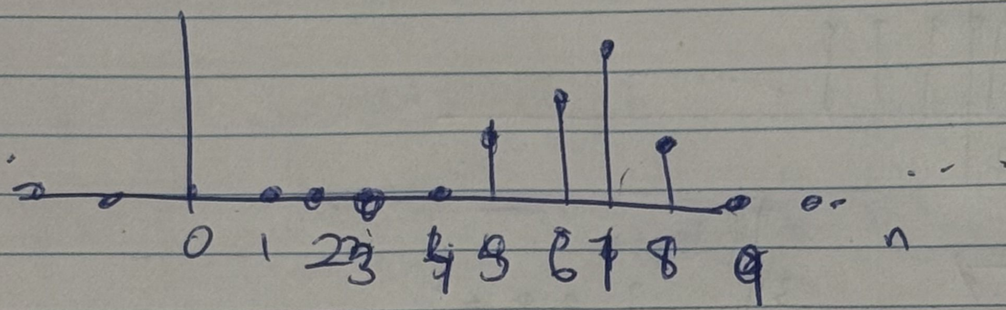
2.29) $h[n] = \begin{cases} 0, & n<0 \\ 1, & n=0,1,2,3 \\ -2, & n=4,5 \\ 0, & n>5 \end{cases}$

(a) $\displaystyle\sum_{k=-\infty}^{\infty} u[k]\, h[n-k] \overset{!}{=} \sum_{0}^{\infty} h[n-k]$

→



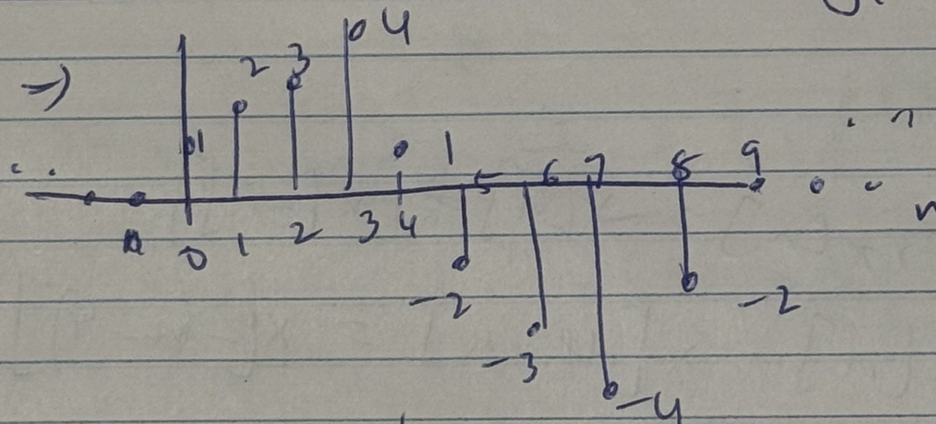
(b) since lti, shifting input shifts output =>



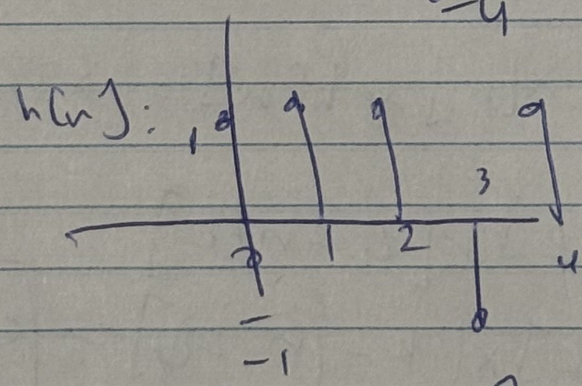
(c) since linear, $y_3[n] = (u[n] - u[n-4]) * h[$

$= y_1[n] - y_2[n]$

=>

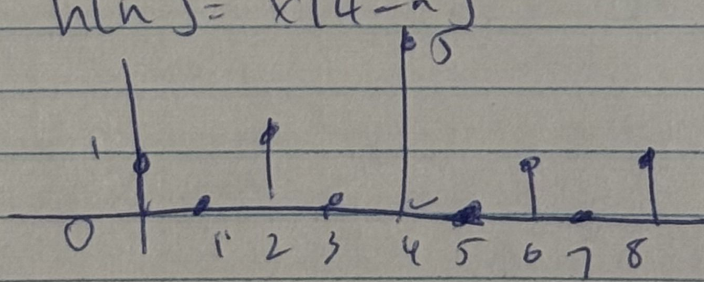


2.35) $h[n]$:

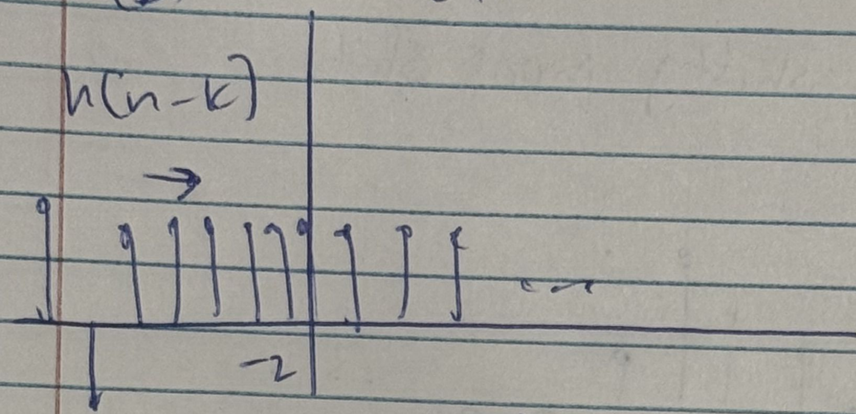


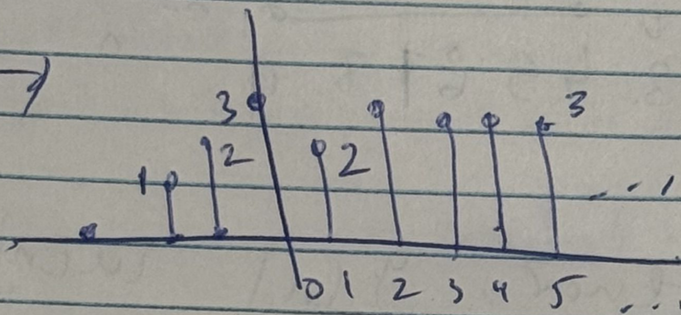
(a) $h[n] = x[4-n]$

=>



(6)





8)

$$c_{xx}[n] = x[n] * [x[-n]]$$

$$h[n] = x[4-n] = x[-(n-4)]$$

$$\rightarrow y[n] = x[n] * h[n] = x[n] * x[4-n]$$

$$= \sum x[k]\, x[4-n+k]$$

$$= \sum_{k} x[k]\, x[k-(n-4)] = \boxed{c_{xx}[n-4]}$$